Homework 2 - Solutions

Derivatives

① $$\frac{d}{dt}\frac{e^t}{e^t+t} = \frac{(e^t+t)(e^t)' - e^t(e^t+t)'}{(e^t+t)^2} \quad \text{by Quotient Rule}$$

$$= \frac{(e^t+t)(e^t) - (e^t)(e^t+1)}{(e^t+t)^2} = \frac{e^t(\cancel{e^t}+t-\cancel{e^t}-1)}{(e^t+t)^2}$$

$$= \boxed{\frac{e^t(t-1)}{(e^t+t)^2}}$$

② $$\frac{d^3}{dx^3}(9-x)^8 = \frac{d^2}{dx^2}\left[8(9-x)^7(-1)\right] = \frac{d^2}{dx^2}\left[-8(9-x)^7\right]$$

$$= \frac{d}{dx}\left[-8\cdot 7(9-x)^6(-1)\right] = \frac{d}{dx}\left[56(9-x)^6\right]$$

$$= 56\cdot 6(9-x)^5(-1) = \boxed{-336(9-x)^5}$$

③ $$g(x,y) = \frac{y^2}{(1+x^2)^3} = y^2(1+x^2)^{-3}$$

$$g_x(x,y) = y^2\frac{d}{dx}(1+x^2)^{-3} = y^2\left[-3(1+x^2)^{-4}(2x)\right] = \boxed{\frac{-6xy^2}{(1+x^2)^4}}$$

$$g_y(x,y) = \frac{1}{(1+x^2)^3}\frac{d}{dy}(y^2) = \frac{1}{(1+x^2)^3}(2y) = \boxed{\frac{2y}{(1+x^2)^3}}$$

④ $f(x,y) = \sin(x^2y^5)$

$$\frac{d}{dx}\sin(x^2y^5) = \cos(x^2y^5)\,\frac{d}{dx}(x^2y^5)$$
$$= \cos(x^2y^5)\,(2x\cdot y^5) = \boxed{2xy^5\cos(x^2y^5)}$$

$$\frac{d}{dy}\sin(x^2y^5) = \cos(x^2y^5)\,\frac{d}{dy}(x^2y^5)$$
$$= \cos(x^2y^5)\,(x^2\cdot 5y^4) = \boxed{5x^2y^4\cos(x^2y^5)}$$

Integration

① $$\int \frac{dx}{1+e^x} = \int \frac{1}{1+e^x}\,dx$$
$$= \int\left(\frac{1+e^x}{1+e^x} - \frac{e^x}{1+e^x}\right)dx$$
$$= \int\left(1 - \frac{e^x}{1+e^x}\right)dx$$

let $u = 1+e^x$

$\frac{du}{dx} = e^x \Rightarrow du = e^x\,dx$

$$= \int 1\,dx - \int \frac{1}{u}\,du$$
$$= x - \ln|u| + C$$
$$= \boxed{x - \ln|1+e^x| + C}$$

(2) $$\int \frac{dx}{x(x^4+1)} = \int \frac{1}{x(x^4+1)}\,dx$$

$$= \int \frac{x^3}{x^4(x^4+1)}$$

let $u = x^4$

$\frac{du}{dx} = 4x^3 \Rightarrow du = 4x^3\,dx$

$$= \frac{1}{4}\int \frac{1}{u(u+1)}\,du$$

$$= \frac{1}{4}\int \left(\frac{1}{u} - \frac{1}{u+1}\right) du$$

by partial fractions

$$= \frac{1}{4}\left(\ln|u| - \ln|u+1|\right) + C$$

$$= \frac{1}{4}\ln\left|\frac{u}{u+1}\right| + C$$

$$= \boxed{\frac{1}{4}\ln\left|\frac{x^4}{x^4+1}\right| + C}$$

(3) $$\int \frac{e^x}{1+e^x}\,dx$$

let $u = 1+e^x$

$du = e^x\,dx$ (as seen in #1)

$$= \int \frac{1}{u}\,du$$

$$= \ln|u| + C$$

$$= \boxed{\ln|1+e^x| + C}$$

④ $\int x(e^x)\,dx$

Integration by Parts

$u = x \qquad v' = e^x$

$u' = 1 \qquad v = e^x$

$= xe^x - \int (1)e^x\,dx$

$= xe^x - \int e^x\,dx = \boxed{xe^x - e^x + C}$